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<p>Let $X(t)$, $t \in R^N$, be a stochastic process assuming values in R^M. The subjects of this investigation are three functionals associated with X: 1) The sojourn time of $X(t)$, $t \in B \subset R^N$, in a set $A \subset R^M$; it is denoted $L(A, B)$. 2) The local time of $X(t)$, which is the density of $L(A, B)$, as a function of sets A, with respect to Lebesgue measure. 3) The random variable $Z(B) = \sup(X(t) : t \in B)$ in the case $M = 1$. The object of the study is to describe specific distributional and structural properties of these functionals, and then formulate conditions on the finite-dimensional distributions of the process which are sufficient for these properties. It was shown how these conditions are related to the structures of classes of stochastic processes commonly arising in theoretical and applied probability.</p>			
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FINAL REPORT

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Measure of the Multiple Self-Intersection Set of a Markov Process

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Abstract

Let $X(t)$, $t \geq 0$, be a Markov process in \mathbb{R}^m with homogeneous transition density $p(t; x, y)$. For a closed bounded set $B \subset \mathbb{R}^m$, X is said to have a self-intersection of order $r \geq 2$ in B if there are distinct points $t_1 < \dots < t_r$ such that $X(t_1) \in B$ and $X(t_j) = X(t_1)$, for $j = 2, \dots, r$. The focus of this work is the Hausdorff measure, suitably defined, of the set of such r -tuples. The main result is that under general conditions on $p(t; x, y)$ as well as the specific condition

$$\int_0^t \sup_{y \in B} \left(\int_B p^r(s; x, y) dy \right)^{1/r} ds < \infty,$$

there is a measure function $M(t)$, defined explicitly in terms of the integral above, such that the corresponding Hausdorff measure of the self-intersection set is positive, with positive probability. The results are applied to Lévy and diffusion processes, and are shown to extend recent results in this area.

1. Introduction and Summary

Let $X(t)$, $t \geq 0$, be a homogeneous Markov process in \mathbb{R}^m , $m \geq 1$, having the transition density function $p(t; x, y)$ representing the density of $X(t)$ at y , conditioned by $X(0) = x$. The focus of this paper is the magnitude of the set of points in the domain at which the sample function intersects itself at least r times, for fixed $r \geq 1$. Many authors have considered the following formulation. Let I_1, \dots, I_r be closed bounded disjoint time intervals, and define the subset of the product set $I_1 \times \dots \times I_r$,

$$S(I_1, \dots, I_r) = \{(t_1, \dots, t_r) : t_j \in I_j, X(t_1) = \dots = X(t_r)\}.$$

Some of the well-known results in this area provide the Hausdorff dimensions of this set for particular kinds of processes, and the current paper provides a contribution in this direction.

Let us recall the elements of Hausdorff measure. Let C be a compact metric space, and let \mathcal{C} be a class of subsets J whose union contains C . For a

Self-intersections and Local Nondeterminism of Gaussian Processes

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AMS Subject Classification (1985): 60G15, 60G17, 60J55

Key words and phrases: Intersections of sample paths, local nondeterminism, local time, Gaussian processes, spectral distribution.

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A CENTRAL LIMIT THEOREM FOR EXTREME SOJOURN TIMES OF STATIONARY GAUSSIAN PROCESSES

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Abstract

Let $X(t)$, $t \geq 0$, be a real measurable stationary Gaussian process with mean 0 and covariance function $r(t)$. For a given measurable function $u(t)$ such that $u(t) \rightarrow \infty$ for $t \rightarrow \infty$, let L_t be the sojourn time of $X(s)$, $0 \leq s \leq t$, above $u(t)$. Assume that the spectral distribution function in the representation of $r(t)$ is absolutely continuous; then $r(t)$ also has the representation $r(t) = \int b(t+s)b(s)ds$, where $b \in L_2$. The main result is: If $b \in L_1$, and if $u(t)$ increases sufficiently slowly, then $(L_t - EL_t)/(\text{Var}(L_t))^{1/2}$ has a limiting standard normal distribution for $t \rightarrow \infty$. The allowable rate of increase of $u(t)$ with t is specified.

1. Introduction and Summary.

Let $X(t)$, $t \geq 0$, be a real measurable stationary Gaussian process with mean 0 and covariance function $r(t) = EX(0)X(t)$. For simplicity we take $r(0) = 1$. For $t > 0$, let $L_t(u)$ be the sojourn time of $X(s)$, $0 \leq s \leq t$, above the level u : $L_t(u) = \text{mes}(s : 0 \leq s \leq t, X(s) > u)$. Then for a given measurable function $u(t)$, we define

$$(1.1) \quad L_t = L_t(u(t)) = \int_0^t 1_{\{X(s) > u(t)\}} ds.$$

The main result of this paper is a new limit theorem for the distribution of L_t , for $t \rightarrow \infty$, where $u(t)$ increases at a specified rate with t . We assume that the spectral distribution function in the representation of $r(t)$ is absolutely continuous. Then $r(t)$ also has the representation (see, for example, [7], page 532),

$$(1.2) \quad r(t) = \int_{-\infty}^{\infty} b(t+s)b(s)ds,$$

where $b(s)$ is the Fourier transform of the square root of the spectral density, and

$$(1.3) \quad \int_{-\infty}^{\infty} |b(s)|^2 ds < \infty.$$

Since $X(t)$ is real valued, $b(s)$ is also real valued. Our main result is:

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LOCAL TIME OF A FUNCTION OF A STOCHASTIC PROCESS

By

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AMS Classification 1985: 60G17, 60J55.

Key words and phrases: Local time, sample function, function of a stochastic process, m th power integrability of local time.

Running head: Local time of function.

POISSON AND EXTREME VALUE LIMIT THEOREMS FOR MARKOV RANDOM FIELDS

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Abstract

Let $X_t, t \in \mathbb{Z}^m$, be a Markov random field assuming values in R^M . Let I_n be a rectangular box in \mathbb{Z}^m with its center at 0 and corner points with coordinates $\pm n$. Let (A_n) be a sequence of measurable subsets of R^M such that $P(X_t \in A_n | X_s, s \in \text{neighborhood of } t) \rightarrow 0$, for $n \rightarrow \infty$; and let $f_n(x)$ be the indicator of A_n . Under appropriate conditions on the nearest-neighbor distributions of (X_t) , the conditional distribution of $\sum_{t \in I_n} f_n(X_t)$, given the values of X_s , for s on the boundary of I_n , converges to the Poisson distribution. An immediate application is an extreme value limit theorem for a real-valued Markov random field.

STATIONARITY; MIXING; EXTREME VALUES; POISSON LIMIT

1. Introduction and summary

The main result of this paper is a limit theorem for the extreme values of a Markov random field on a discrete lattice. Let $X_t, t \in T$, be a family of real random variables on some probability space, and where T is a countable index set. Let $I_n, n \geq 1$, be a sequence of finite subsets of T such that $I_n \subset I_{n+1}$ and $T = \bigcup_{n \geq 1} I_n$. Define

$$(1.1) \quad M_n = \max_{t \in I_n} X_t.$$

The extreme value limit problem is as follows: we seek conditions under which there exist a non-degenerate distribution function $G(x)$ and real sequences (a_n) and (b_n) , with $a_n > 0$ such that

$$(1.2) \quad \lim_{n \rightarrow \infty} P(a_n^{-1}(M_n - b_n) \leq x) = G(x)$$

at all continuity points x . When the X_t 's are independent with a common

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SOJOURN TIMES IN A CONE FOR A CLASS OF VECTOR GAUSSIAN PROCESSES*

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Abstract. Let $X(t)$ be a Gaussian process in R^m with stationary increments. Let $S(t)$ be the covariance matrix of $X(t) - X(0)$, and assume that there is a positive real function $\sigma^2(t)$ and a positive definite matrix R such that $S(t) \sim \sigma^2(t)R$, for $t \rightarrow 0$. Let $f(x)$, $x \in R^m$, be a real bounded Borel function such that $f(cx) = f(x)$, for all $c > 0$. The main result is that $t^{-1} \int_0^t f(X(s) - X(0)) ds$ has, for $t \rightarrow 0$, a limiting distribution, which is identified. A particular case of this is a new result for the limiting uniform distribution of the positive sojourn time proportion for $m = 1$. Another application is a limit theorem for the conditional distribution of the sojourn time proportion above a high level.

Key words. Gaussian vector process, stationary increments, sojourn time, local time, high level, cone, slow variation, tightness, weak convergence

AMS(MOS) subject classifications. 60G15, 60G10

1. Introduction and summary. Let K be a cone in R^m , that is, a Borel set such that $x \in K$ implies $cx \in K$ for every $c > 0$. Let $X(t)$, $t \geq 0$, be a measurable stochastic process assuming values in R^m . The main result of this paper is concerned with the limiting distribution of the proportion of time spent in K by $X(s) - X(0)$, $0 \leq s \leq t$, for $t \rightarrow 0$, in the case where $X(\cdot)$ belongs to a particular class of Gaussian processes.

Let $X(t)$, $t \geq 0$, be a measurable Gaussian process in R^m with mean 0 and stationary increments. For any vector x , the notation x' will be used for the transpose. Put

$$(1.1) \quad S(t) = E[X(t) - X(0)][X(t) - X(0)]'.$$

Assume that $S(t)$ is continuous for $t \geq 0$, positive definite for $t > 0$, and $S(0) = 0$ -matrix. Our main assumption about S is the following hypothesis.

Hypothesis. There is a positive definite matrix R and a continuous positive function $\sigma^2(t)$, $t > 0$, which is slowly varying for $t \rightarrow 0$, such that

$$(1.2) \quad S(t)/\sigma^2(t) \rightarrow R \quad \text{for } t \rightarrow 0.$$

Here $S(t)/\sigma^2(t)$ is the matrix S times the scalar $1/\sigma^2$. Recall that a function σ^2 is slowly varying if $\sigma^2(tx)/\sigma^2(t) \rightarrow 1$, for $t \rightarrow 0$, for every $x > 0$.

Let $f(x)$, $x \in R^m$, be a real bounded Borel function. It is called a *cone function* if

$$(1.3) \quad f(cx) = f(x) \quad \text{for all } c > 0, x \in R^m.$$

The indicator of a cone in R^m is obviously a cone function.

While the main theorem is of interest for indicators of cones it is stated more generally for cone functions.

THEOREM 1. Let Z be a normal random vector with mean 0 and covariance matrix R , and let $\phi_R(z)$, $z \in R^m$, be the corresponding normal density. Under the conditions on $X(t)$ and $S(t)$ stated above, if f is a cone function, then

$$(1.4) \quad t^{-1} \int_0^t f(X(s) - X(0)) ds$$

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The Modulator of the Local Time

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Abstract

Let $x(t)$, $0 \leq t \leq 1$, be a real measurable function having a local time $\alpha(x, t)$ which is a continuous function of t for almost all x . It is also assumed that, for some $m \geq 2$ and some real interval B , $\alpha^m(x, 1)$ is integrable over B . The modulator is a function $M_m(t, B)$, $t > 0$, defined in terms of α . It is shown that the modulator serves as a measure of the smoothness of the $L_m(B)$ -valued function $\alpha(\cdot, t)$ with respect to t . Then it is shown that the modulator plays a central role in precisely describing certain irregularity properties of $x(t)$. The results are applied to the case where $x(t)$ is the sample function of a real stochastic process. In this way new results are obtained for large classes of Gaussian and Markov processes.

1. Introduction and Summary

Let $x(t)$, $0 \leq t \leq 1$, be a real measurable function. For every pair of linear Borel sets A and I , $I \subset [0, 1]$, define $\nu(A, I) = \text{Lebesgue measure } \{t: t \in I, x(t) \in A\}$. If, for fixed I , $\nu(\cdot, I)$ is absolutely continuous as a measure of sets A , then its Radon-Nikodym derivative, which we denote by $\alpha_I(x)$, is called the local time of $x(t)$, $t \in I$. It satisfies

$$(1.1) \quad \nu(A, I) = \int_A \alpha_I(x) dx.$$

Define

$$(1.2) \quad \alpha(x, t) = \alpha_{[0, t]}(x).$$

In [3] and [6] we proved that the function $x(t)$ has specified irregularity properties under the hypothesis of "temporal continuity" of the local time, which means that

$$(1.3) \quad \alpha(x, t) \text{ is continuous in } t \text{ for almost all } x.$$

In the present paper we prove additional irregularity properties of $x(t)$ under the additional assumption

$$(1.4) \quad \int_B \alpha^m(x, 1) dx < \infty$$

for some real interval B and some integer $m \geq 2$. The assumption (1.4) implies

$$\int_B \alpha^m(x, t) dx < \infty$$

for all $0 \leq t \leq 1$ because $\alpha(x, t)$ may be assumed to be monotonic in t (see [1]).

Sojourns and Extremes of a Stochastic Process Defined as a Random Linear Combination of Arbitrary Functions

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ABSTRACT

Let X_t be a real stochastic process of the form $(X, f(t))$, where X is a random vector in R^n with an orthogonally invariant distribution, and $f(t), 0 \leq t \leq 1$, assumes values in R^n . Put $x_0 = \sup \{x: P(\|X\| \leq x) < 1\}$, finite or infinite. For real $u < x_0$, let L_u be the sojourn time of $X_t, 0 \leq t \leq 1$, above u . The main results are limit theorems for the distribution of L_u and for $P(\max_t X_t > u)$, for $u \rightarrow x_0$. The hypotheses are stated in terms of the conditions on the tail of the distribution of X which are used in extreme value theory for i.i.d. random variables.

1. INTRODUCTION AND SUMMARY

The subject of this paper is the study of certain sojourn time and extreme value limit theorems for a particular class of real stochastic processes. Let $X = (X_1, \dots, X_n)$ be real random vector in R^n and $f(t) = (f_1(t), \dots, f_n(t)), 0 \leq t \leq 1$, a real vector valued function. Then,

$$X_t = (X, f(t)) = \sum_{j=1}^n X_j f_j(t), \quad 0 \leq t \leq 1, \quad (1.1)$$

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AMS Subject Classification 1985 Primary 60G17, Secondary 60K99.

Key words and phrases: Sojourns of a process, extremes of a process, distribution of norm, orthogonal invariance, domain of attraction, extreme value distribution.

**THE MAXIMUM OF A GAUSSIAN PROCESS WITH NONCONSTANT
VARIANCE: A SHARP BOUND FOR THE DISTRIBUTION TAIL**

by

SIMEON M. BERMAN and NORIO KÔNO

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Local Times of Stochastic Processes with Applications to
the Sample Functions of Markov Processes

by

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process, approximate local growth, measure of level set,
 γ -measure variation

Running head: Local Times of Stochastic Processes

THE SUPREMUM OF A PROCESS WITH STATIONARY INDEPENDENT AND SYMMETRIC INCREMENTS

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Let $X_t, t \geq 0$, be a process with stationary independent and symmetric increments. If the tail of the Lévy spectral measure in the representation of the characteristic function is of regular variation of index $-\alpha$, for some $0 < \alpha < 2$, then $P(\sup(X_s: 0 \leq s \leq t) > u) \sim P(X_t > u)$, for $u \rightarrow \infty$, for each $t > 0$.

AMS (1985) Subject Classification: 60J30, 60F10.

Independent increments • supremum distribution • regular variation • sojourn above high level

1. Introduction and summary

Let $X_t, t \geq 0$, be a separable stochastic process with independent increments, which is centered and has no fixed points of discontinuity. Then there is a version of the process which, with probability 1, has sample functions which are locally bounded and for which $\sup(X_s: 0 \leq s \leq t)$ is a well defined random variable. The main result of this paper is the asymptotic relation

$$P(\sup(X_s: 0 \leq s \leq t) > u) \sim P(X_t > u) \quad \text{for } u \rightarrow \infty, \quad (1.1)$$

for a large class of such processes.

Suppose that the increments are stationary. Then, by the classical result of Lévy, the characteristic function of $X_t - X_s$, for $0 \leq s < t$ is of the form

$$E e^{i\theta(X_t - X_s)} = e^{-(t-s)f(\theta)},$$

where $e^{-f(\theta)}$ is the characteristic function of an infinitely divisible distribution. In the particular case which we will consider here, where the increments are symmetrically distributed about 0, the function $f(\theta)$ takes the form

$$f(\theta) = 2 \int_0^\infty (1 - \cos \theta x) \frac{1+x^2}{x^2} dG(x), \quad (1.2)$$

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SPECTRAL CONDITIONS FOR LOCAL NONDETERMINISM

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Let $X(t)$ be a real Gaussian process with stationary increments and spectral distribution function $F(x)$. Put $\phi(t) = F(\infty) - F(1/t)$. Sufficient conditions in terms of F are given for the process to be locally ϕ -nondeterministic. These are formulated for discrete and absolutely continuous functions F . The results in the discrete case are applied to the analysis of the local time of a random Fourier series with i.i.d. coefficients. The class of distributions of the coefficients includes not only the normal distribution but others such as the symmetric stable distribution.

AMS (1985) Subject Classifications: 60G10, 60G15, 60J55.

local nondeterminism • local time • Gaussian process • stationarity • spectral distribution • random Fourier series

1. Introduction and summary

Let $X(t)$, $t \geq 0$, be a separable Gaussian process with mean 0, and let J be an open interval on the t -axis. Assume that there exists $d > 0$ such that

$$\begin{aligned} EX^2(t) &> 0, \quad t \in J, \quad \text{and} \\ E(X(t) - X(s))^2 &> 0 \quad \text{for } 0 < |t - s| < d, \quad s, t \in J. \end{aligned} \quad (1.1)$$

The concept of local nondeterminism (LND) was introduced by the author in [2]. According to Lemmas 2.1 and 2.2 of that paper, the definition of LND is equivalent to the following: For every $m \geq 2$, let $t_1 < t_2 < \dots < t_m$ be variable ordered points in J ; then the determinant of the covariance matrix of the m standardized random variables,

$$\frac{X(t_1)}{(\text{Var } X(t_1))^{1/2}}, \frac{X(t_2) - X(t_1)}{(\text{Var}(X(t_2) - X(t_1)))^{1/2}}, \dots, \frac{X(t_m) - X(t_{m-1})}{(\text{Var}(X(t_m) - X(t_{m-1})))^{1/2}}$$

is, as a function of $t_1 < \dots < t_m$, bounded away from 0. The concept was extended by Cuzick [4], who defined local ϕ -nondeterminism, LND(ϕ), by replacing the

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EXTREME SOJOURNS OF DIFFUSION PROCESSES

by

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